Physics 3310
Chapter 5
NOTES

Circular Motion; Gravitation

KEY!

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STUDENT NAME

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TEACHER'S NAME

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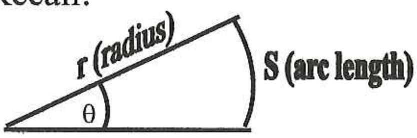
PERIOD

CHAPTER 5 - Circular Motion; Universal Gravitation

Circular Motion is a class of ***PERIODIC MOTION***. Motion that repeats itself; that completes a cycle; that finishes where it starts; that oscillates; motion that repeats itself; that completes a cycle; that finishes where it starts; that oscillates; motion that repeats itself; that completes a cycle; that finishes where it starts; that oscillates…

Recall:



$$\theta \equiv \frac{s}{r} \text{ (radians)}$$ $\Rightarrow s = r\theta$

Conversions:

$$\left(\angle^{o}\right)\left(\frac{\pi}{180^{o}}\right) = \angle_{radians} \quad \text{eg.} \quad \left(75^{o}\right)\left(\frac{\pi}{180^{o}}\right) = 1.31_{radians}$$

$$\left(\angle_{radians}\right)\left(\frac{180^{o}}{\pi}\right) = \angle^{o} \quad \text{eg.} \quad \left(3_{rad}\right)\left(\frac{180^{o}}{\pi}\right) = 172^{o}$$

Uniform Circular Motion

- ☑ Uniform in this case means ***constant speed.***
- ☑ Can an object moving with a ***constant speed*** have acceleration?

  Yes! acceleration is change in velocity which is speed with direction. so changing direction = acceleration!

- ☑ If so, we physicists are going to want to find a value for it, but how?

NOTE: The magnitude of the velocity is not changing, but the direction is changing.

so we have acceleration even though its constant speed.

In Figure 2 triangle made by the two radii and Δl will be similar to the triangle made by v_1 ,v_2 , Δv.

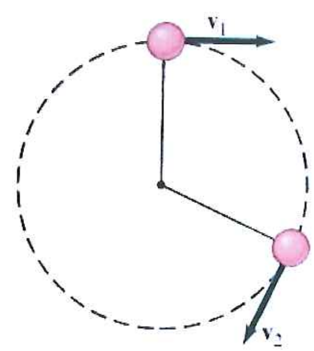


Figure 1 a small object moving in a circle. Note the instantaneous velocity is always tangent to the circular path.

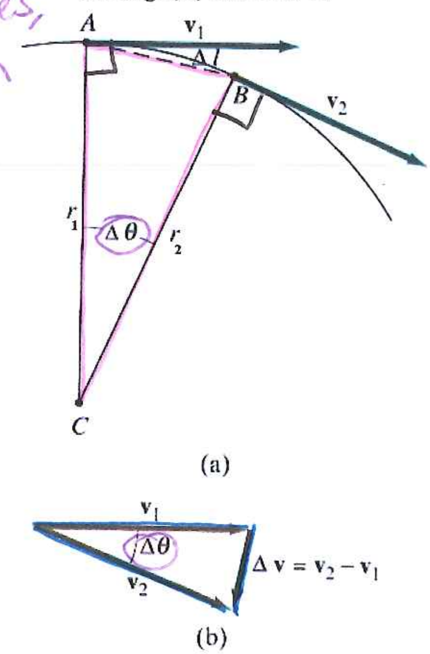
PROOF:

By definition:

The angle between the radius and the instantaneous velocity is always 90°. (See figure 1)

For this to remain true, the angle between r_1 and r_2 must be equal to the angle between v_1 and v_2. (See figure 2 (a) and (b)) Since they are both isosceles triangles, and $\Delta\Theta$ is equivalent, then the other two ∡'s are equivalent ⇒ they are similar triangles.

Figure 2 Determining the change in the velocity, Δv, for a particle moving in a circle. The length Δl is the cord from A to B. As $\Delta\theta$ approaches zero, Δl approaches the arc length, s, from A to B.



Therefore, $\dfrac{\Delta v}{v} = \dfrac{\Delta l}{r}$

b/c they are similar triangles, corresponding sides are proportional.

And $\boxed{\Delta v = \dfrac{v}{r}\Delta l}$

If $a = \dfrac{\Delta v}{\Delta t}$ old def. then $a = \dfrac{v}{r}\dfrac{\Delta l}{\Delta t}$

Note: For small values of $\Delta\theta$, Δl is $\approx$ s (arc length).
See Figure 2

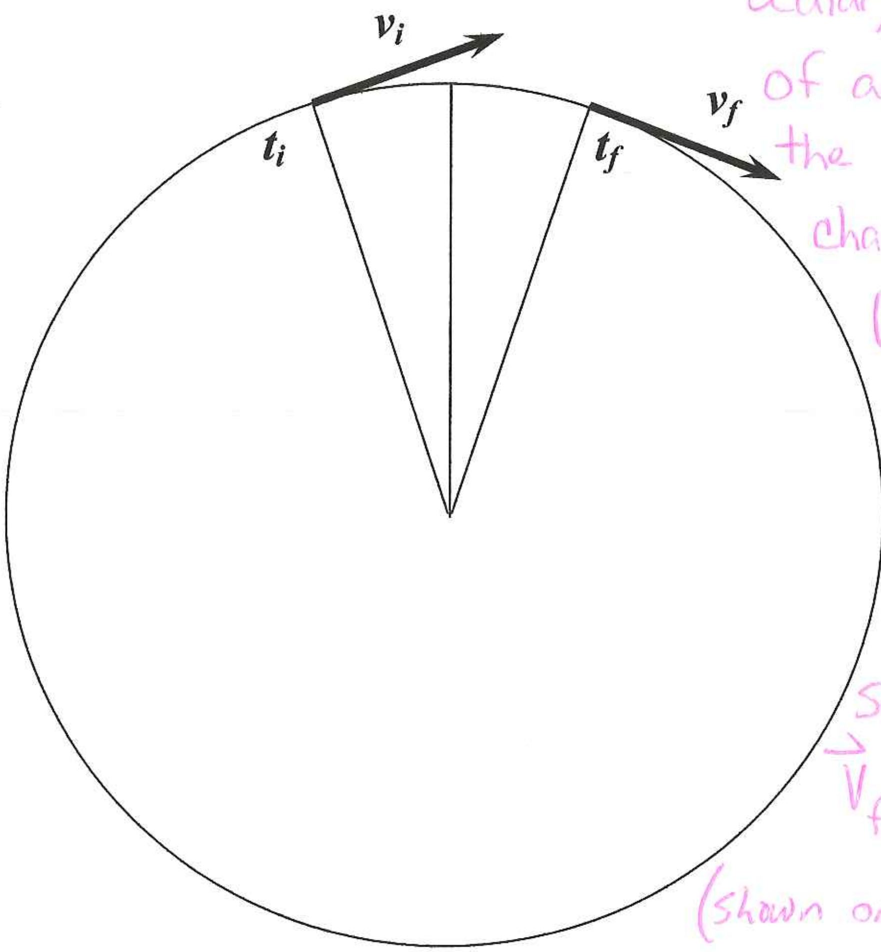
Therefore since $\dfrac{\Delta l}{\Delta t} \approx \dfrac{\Delta s}{\Delta t}$, and $\dfrac{\Delta s}{\Delta t}$ is our definition of linear speed, v. $\left(\vec{v} = \dfrac{\Delta x}{\Delta t}\right)$

Then $\boxed{a = \dfrac{v^2}{r}}$ (***Equation 1)***

Hopefully, it is clear that this formula can only give positive answers, and therefore only can find the ***magnitude of the acceleration***. Acceleration is a VECTOR so… in ***what direction is the acceleration***?

PROOF: $\vec{a} = \frac{\Delta\vec{v}}{t} = \frac{\vec{v}_f - \vec{v}_i}{t}$

Since time is a scalar, the direction of acceleration is the direction of the change in velocity (which is a vector).



the change in velocity is found using vector subtraction.

$\vec{v}_f + (-\vec{v}_i)$

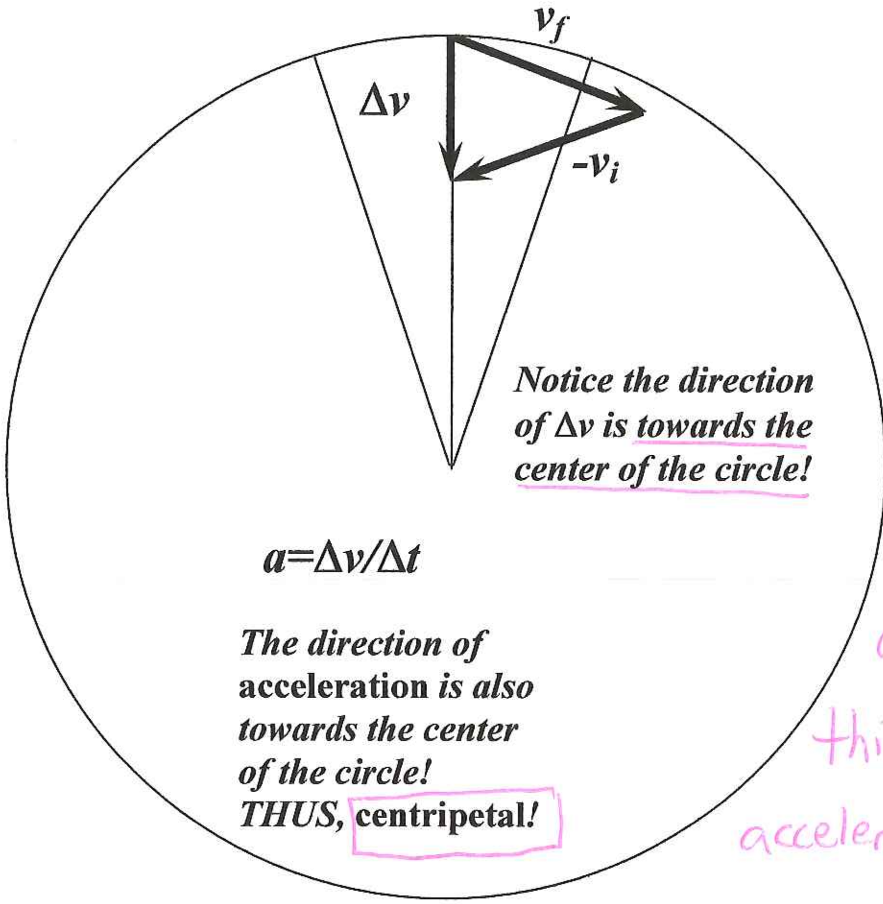
(shown on next page)

Centripetal vs. Centrifugal

Centri from the Latin Centrum: the middle point of a circle, the center.
Petal from the Latin Peto – to seek, strive after, endeavor to obtain.
Fugal from the Latin Fugito – to flee, to fly from, avoid or shun.

By looking at the *direction* of the change in velocity, you should be able to see the *direction* of the acceleration.

Is it Centripetal or Centrifugal?



∴ the acceleration is towards the center. we call this centripetal acceleration, $\vec{a}_c$.

The acceleration of an object moving with Uniform Circular Motion is known as *centripetal acceleration* or *radial acceleration.*

To use ***equation # 1*** we will need to know both the radius of the circle and the speed of the object.

$$\boxed{a = \frac{v^2}{r}}$$

Problems:

1. (I) A jet plane traveling 1800 km/h (500 m/s) pulls out of a dive by moving in an arc of radius 6.00 km. What is the plane's acceleration in g's?

$v = 500\,m/s$

$r = 6km = 6000m$

$a_c = \frac{v^2}{r} = \frac{500\,m/s^2}{6000\,m}$

$a_c = 41.\overline{66}\,m/s^2 \times \frac{1g}{9.8\,m/s^2}$

$\boxed{a_c = 4.252g}$

2. (I) Calculate the centripetal acceleration of the Earth in its orbit around the Sun and the net force exerted on the Earth. What exerts this force on the Earth? Assume that the Earth's orbit is a circle of radius 1.50×10^{11} m.

$r = 1.5 \times 10^{11}\,m$

$a_c = \frac{v^2}{r} = \frac{(29885.77\,m/s)^2}{1.5 \times 10^{11}\,m} = \boxed{.00595\,m/s^2}$

$v = \frac{distance}{time} = \frac{2\pi r}{T}$

$= \frac{2\pi(1.5 \times 10^{11}\,m)}{365\cancel{days} \cdot \frac{24\cancel{hr}}{1\cancel{day}} \cdot \frac{3600s}{1\cancel{hr}}}$

$v = 29885.77\,m/s$

now to find the force, $F = ma$ and mass is $5.97 \times 10^{24}\,kg$ (from formula sheet)

$F = 5.97 \times 10^{24}\,kg(.00595\,m/s^2)$

$\boxed{F = 3.555 \times 10^{22}\,N}$

3. Will the acceleration of a car be the same when it travels around a sharp curve at 60 km/h, as when it travels around a gentle curve at the same speed? Explain.

$r_{sharp} < r_{gentle}$ and since $a_c = \frac{v^2}{r}$

$a = \frac{v^2}{r}$

a_c will be greater when it is a sharp curve.

sharp curve: gentle curve

How do we find the speed of an object going in a circle? → $V = \frac{d}{t} = \frac{\text{Circumference}}{\text{Period}}$

☑ Measure the time it takes for the object to complete a certain number of revolutions. $= \frac{2\pi r}{T}$

EXAMPLE: We know an object makes 10 revolutions in 2 seconds. With the information given we know the frequency of each revolution.

$$f = \frac{10_{rev}}{2_{\text{sec}}} = 5_{\frac{rev}{\text{sec}}}$$

(frequency is often measured in Hertz (Hz))

The time needed to complete one revolution is known as the Period (T).

$$T \equiv \frac{1}{f} \text{ and } f \equiv \frac{1}{T} \quad \therefore \quad T = \frac{1}{5_{\frac{rev}{\text{sec}}}} \text{ or } T = 0.2_{\frac{\text{sec}}{rev}}$$

(Period is measured in seconds, minutes, hours, etc.)

How far does an object move in one revolution? (circumference)

1 rev $=2\pi r$

(***Equation 2)***

$$\therefore \boxed{v = \frac{distance}{time} = \frac{2\pi r}{T}}$$

More Useful Formulas…

If we substitute ***Equation 2*** into ***Equation 1***…

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{\frac{4\pi^2 r^{\cancel{2}}}{T^{\cancel{2}}}}{\cancel{r}} \times \frac{\frac{1}{\cancel{r}}}{\frac{1}{\cancel{r}}}$$

THEN SIMPLIFY

$$\boxed{a_c = \frac{4\pi^2 r}{T^2}}$$ ***(Equation 3)***

You don't need to know the speed to find a_c with this formula! Look back at page 6 number 2:

$$a_c = \frac{4\pi^2(1.5\times10^{11})}{\left[365 \times \frac{24}{1} \times \frac{3600}{1}\right]^2} = .00595\ m/s^2$$

Same answer!

COMBINE THE OLD WITH THE NEW

Nonuniform Circular Motion → now we aren't assuming constant speed.

The total acceleration of a body is the tangential (linear) acceleration plus the radial (centripetal) acceleration.

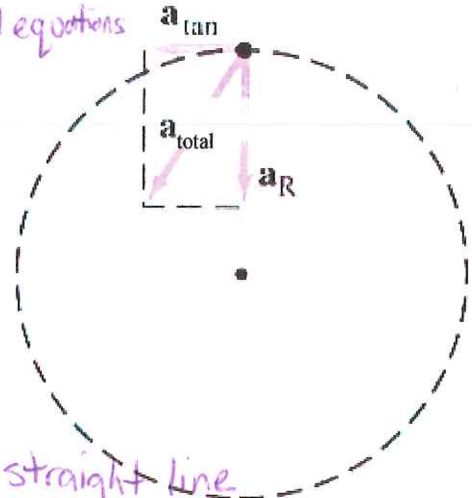
$$\boxed{\vec{a}_{Total} = \vec{a}_{tangential} + \vec{a}_{radial}}$$

OR *(in relation to the velocity)*

$$\vec{a}_{Total} = \vec{a}_{\parallel} + \vec{a}_{\perp}$$

a from constant accel equations

$a_c = \frac{v^2}{r}$



* remember these are vector quantities, so you need to do vector addition

* $a_{\parallel}$ is like taking the circle and bending into straight line

4. (I) A racing car starts from ***rest*** in the pit area and accelerates at a uniform rate to a speed of **30 m/s** in **11 s**, moving on a circular track of radius **500m**. Determine the tangential and centripetal components of the total acceleration exerted on the car (by the ground).

$v_i = 0\,m/s$

$v_f = 30\,m/s$

$t = 11\,sec$

$r = 500\,m$

$a_{\parallel}$: $v_f = v_i + at$

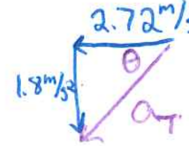
$30\,m/s = a(11\,sec)$

$\boxed{a = 2.72\,m/s^2}$

$a_{\perp}$: $a_c = \frac{v^2}{r} = \frac{30^2}{500}$

$\boxed{a_c = 1.8\,m/s^2}$

* if asked for total $\vec{a}$, add the two components.



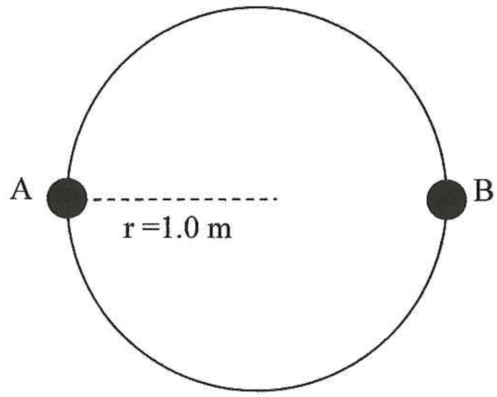
$a_T = \sqrt{2.72^2 + 1.8^2} = 3.27\,m/s^2$

$\theta = \tan^{-1}\left(\frac{1.8}{2.72}\right) = 33.42°$

$\boxed{a_T = 3.27\,m/s^2 \text{ @ } 33.42°}$

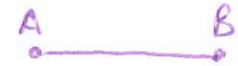
EXAMPLE PROBLEM: Nonuniform circular motion

A particle moves clockwise in a circle of radius 1.0 m. It starts at rest at the origin at time, t=0. Its speed increases at a constant rate of 1.6 m/s^2. See Figure below.



A. How long does it take to travel half way around the circle (point A to Point B)?

think of it being straight line



$V_i = 0 \, m/s$

$a = 1.6 \, m/s^2$

$\Delta x = \frac{1}{2}(2\pi r) = \pi$

$\Delta x = V_i t + \frac{1}{2} a t^2$

$\pi = 0 + \frac{1}{2}(1.6)t^2$

$\boxed{t = 1.98 \text{sec}}$

B. What is its speed at point B?

$V_i = 0 \, m/s$

$a = 1.6 \, m/s^2$

$t = 1.98 s$

$V_f = V_i + at$

$= 1.6(1.98) \Rightarrow \boxed{V_f = 3.17 \, m/s}$

C. What is the direction of its velocity at that time?

Velocity is tangent to the radius so South or $-\hat{y}$

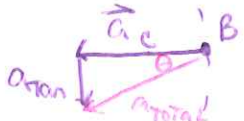
D. What is the centripetal (radial) acceleration?

$\vec{a}_c = \frac{v^2}{r} = \frac{3.17^2}{1} = 10.058 \, m/s^2$ radially inward $(-\hat{x})$

E. What is the Tangential (linear) acceleration?

$\vec{a}_{tan} = -1.6 \, m/s^2 \, \hat{y}$

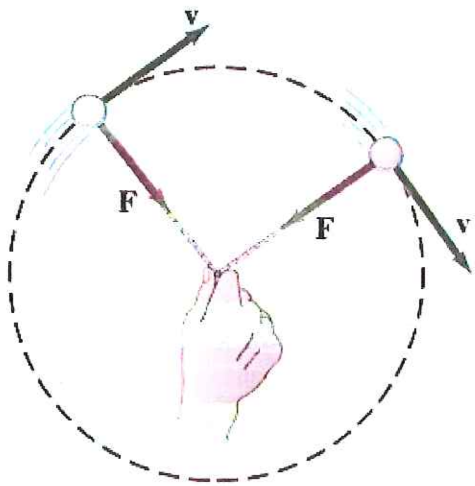
F. What is the magnitude and direction of the total acceleration at point B?



$a_{TOTAL} = \sqrt{10.058 \, m/s^2{}^2 + 1.6 \, m/s^2{}^2} = \boxed{10.179 \, m/s^2}$

$\theta = \tan^{-1}\left(\frac{1.6}{10.058}\right) = \boxed{9.038°}$

DYNAMICS of Circular Motion



Revisiting Newton's 2nd Law

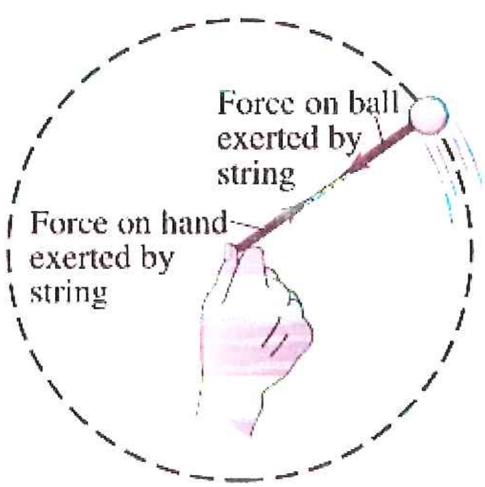
Newton stated that if a body is accelerating, there must be a NET force acting on it, in the direction of the acceleration.

The NET force the cause circular motion is called the CENTRIPETAL force.

Note: This is not a new type of force, we will treat it very much the same as we did any NET force.

$$\sum F = ma \quad \text{and} \quad \boxed{F_c = ma_c = \frac{mv^2}{r} = \frac{m4\pi^2 r}{T^2}}$$

A **centripetal** (center seeking) force causes circular motion NOT a centrifugal (center fleeing) force.

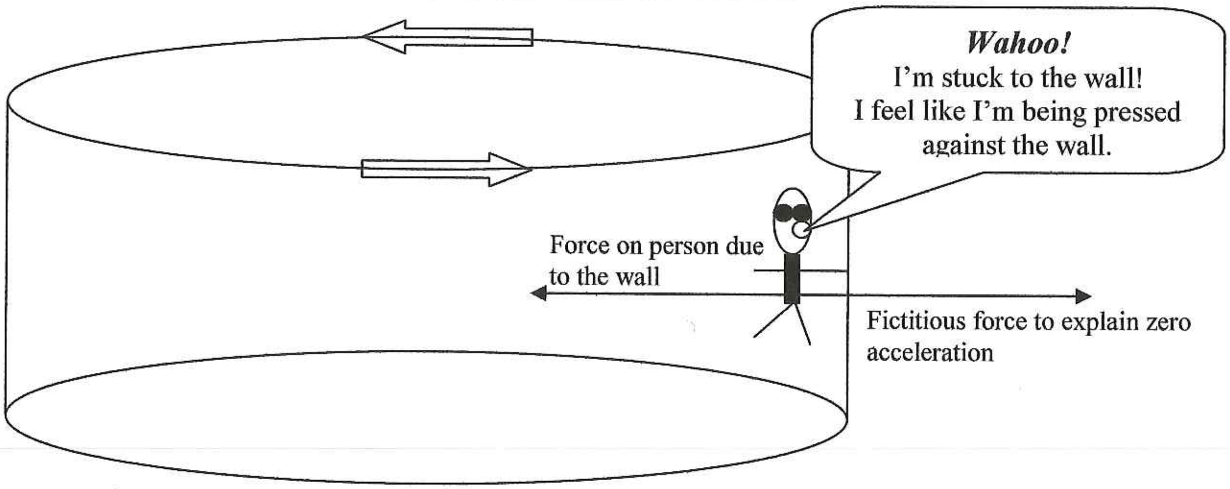


- The force that is acting on the ball causing circular motion is inward (centripetal).
- The force acting on the hand is outward, and NOT causing circular motion.
- A centrifugal force is a "fictitious" or "pseudo" force caused by viewing the situation from an accelerating reference

frame.

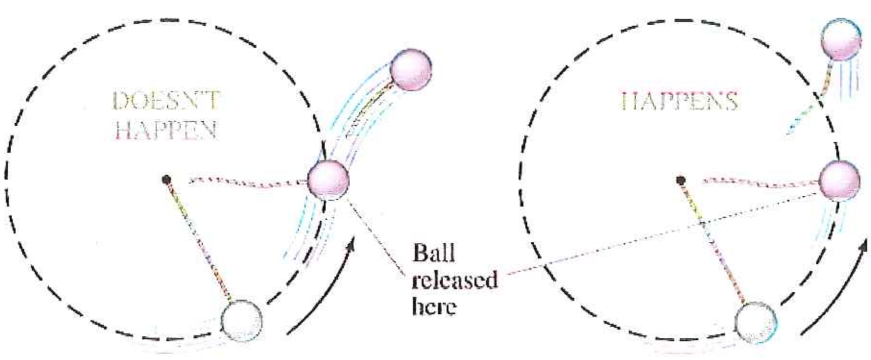
A "fictitious" force is real to an observer in an accelerating frame of reference. SEE CHAPTER 4 NOTES → non-inertial reference frames
EXAMPLE: Person in a rotating reference frame.

THE ROTOR RIDE



- In the accelerating reference frame (i.e. the person rotating around with the wall) the person is at rest and not moving. This person can't use Newton's Laws to explain why he is at rest. He must introduce a "fictitious" force (centrifugal force) which is quite real to him. He is in a non-inertial (accelerating) reference frame.

Why is it said that centrifugal "fictitious" force is a misconception, and NOT real?



(a) (b)

- Because to an observer in an inertial reference frame "fictitious" force does <u>not</u> exist.

- Remember, Newton's laws only apply in inertial reference frames.

- Centrifugal (fictitious) force is a non-Newtonian, and is caused by acceleration and is only real to an observer in the accelerating reference frame.

- In the picture above (a) represents what would happen if centrifugal forces were real. (b) Shows what **does** happen in an inertial frame of reference.

Problems:

4. (I) A horizontal force of 280 N is exerted on a 2.0-kg discus as it is rotated uniformly in a horizontal circle (at arms length) of radius 1.00 m. Calculate the speed of the discus.

$F_c = 280N$

$m = 2kg$

$r = 1.00m$

$v = ?$

$F_c = \frac{mv^2}{r}$

$280N = \frac{(2kg)v^2}{1.0m}$

$\boxed{v = 11.832 m/s}$

5. (II) A flat puck (mass M) is rotated in a circle on a frictionless air hockey tabletop, and is held in this orbit by a light cord which is connected to a dangling mass (mass m) through the central hole as shown in Fig. 5-32. Show that the speed of the puck is given by $v = \sqrt{\frac{mgr}{M}}$.

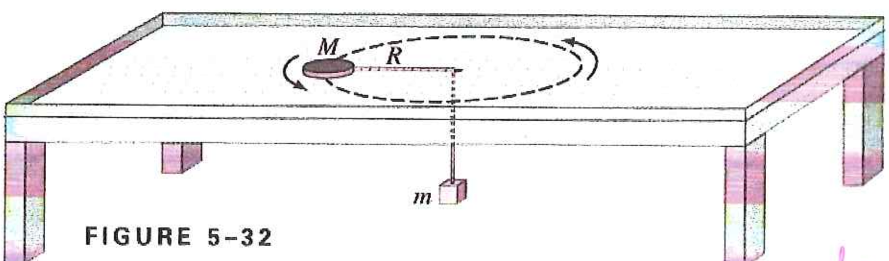


FIGURE 5-32

Great Problem!

We know $F_c = \frac{mv^2}{r}$.

In this situation, F_c is due to the tension in the String, which is due to the weight of the hanging mass.

This means $F_c = F_g = mg = \frac{Mv^2}{r}$ ← solve for v!

$mgr = Mv^2$

$\frac{mgr}{M} = v^2 \Rightarrow v = \sqrt{\frac{mgr}{M}}$ ✓✓

6. (II) A 0.40-kg ball, attached to the end of a horizontal cord, is rotated in a circle of radius 1.3 m on a frictionless horizontal surface. If the cord will break when the tension in it exceeds 60 N, what is the maximum speed the ball can have? How would your answer be affected if there were friction?

$m = .40kg$

$r = 1.3m$

$F_c = 60N$

$v = ?$

$F_c = \frac{mv^2}{r}$

$60 = \frac{.4(v^2)}{1.3}$

$\boxed{v = 13.96 m/s}$

friction would not affect this answer.

7. (II) A ball on the end of a string is cleverly revolved at a uniform rate in a vertical circle of radius 85.0 cm, as shown in Fig. 5-33. If its speed is 4.15 m/s and its mass is 0.300 kg, calculate the tension in the string when the ball is (a) at the top of its path, and (b) at the bottom of its path.

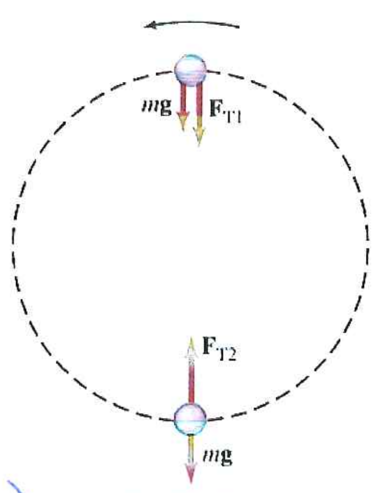


FIGURE 5-33

$r = .85m$

$v = 4.15 m/s$

$m = .30 kg$

$F_T = ?$

a. $F_c = F_T + F_g = \frac{mv^2}{r}$

$F_T = \frac{mv^2}{r} - mg$

$= \frac{.3(4.15)^2}{.85} - .3(9.8)$

$\boxed{F_T = 3.139N}$

b. $F_c = F_T - F_g = \frac{mv^2}{r}$

$F_T = \frac{mv^2}{r} + mg$

$= \frac{.3(4.15)^2}{.85} + .3(9.8) \rightarrow \boxed{F_T = 9.0185N}$

8. (II) A device for training astronauts and jet fighter pilots is designed to rotate the trainee in a horizontal circle of radius 10.0 m. If the force felt by the trainee is 7.75 times her own weight, how fast is she rotating? Express your answer in both m/s and rev/s.

$r = 10.0\text{m}$

$F_c = 7.75mg$

$v = ?$

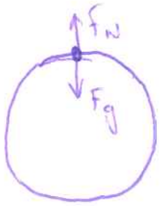
$$F_c = \frac{mv^2}{r}$$

$$7.75\cancel{m}g = \frac{\cancel{m}v^2}{10} \rightarrow \boxed{v = 27.559 \text{ m/s}}$$

$$\frac{27.559\text{m}}{\text{sec}} \times \frac{1\text{ rev}}{2\pi(10)\text{m}} = \boxed{.4386 \text{ rev/sec}}$$

9. (II) How many revolutions per minute would a 15-m-diameter Ferris wheel need to make for the passengers to feel "weightless" at the topmost point of the trip?

$r = 15\text{m}$



"weightless" when $F_N = 0\text{N}$, so $F_c = F_g = mg$

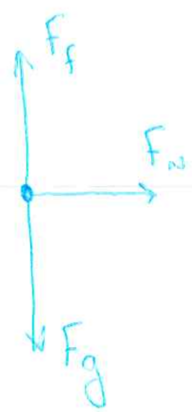
$$F_c = \frac{mv^2}{r}$$

$$\cancel{m}g = \frac{\cancel{m}v^2}{r}$$

$$9.8 = \frac{v^2}{15} \rightarrow v = 12.12 \text{ m/s}$$

$$\frac{12.12\text{m}}{\text{sec}} \times \frac{60\text{sec}}{1\text{min}} \times \frac{1\text{rev}}{2\pi(15)\text{ m}} = \boxed{7.719 \text{ rev/min}}$$

10. (II) In a "Rotor-ride" at a carnival, people pay money to be rotated in a vertical cylindrically walled "room." (Refer to note packet for picture) If the room radius is 5.0 m, and the rotation frequency is 0.50 revolutions per second when the floor drops out, what is the minimum coefficient of static friction so that the people will not slip down? People describe this ride by saying they were being "pressed against the wall." Is this true? Is there really an outward force pressing them against the wall? If so, what is its source? If not, what is the proper description of their situation (besides "scary")?
[Hint: First draw the free-body diagram for a person.]



$r = 5\,m$

$f = .5 \frac{rev}{sec}$

$T = \frac{1}{f} = 2\,sec$

So $v = \frac{2\pi r}{T} = \frac{2\pi(5)}{2} = 15.708\,m/s$

$F_c = F_N = \frac{mv^2}{r}$

BUT WHATS F_N?

we know $F_f = \mu F_N$ and

here $F_f = F_g = \mu F_N$

So $mg = \mu F_N \Rightarrow \boxed{F_N = \frac{mg}{\mu}}$

$\frac{\cancel{m}g}{\mu} = \frac{\cancel{m}v^2}{r}$

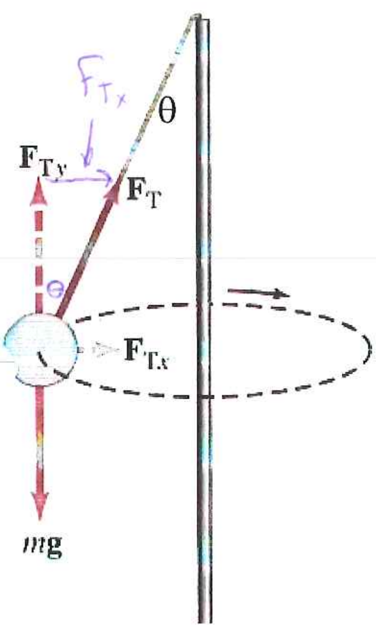
$\frac{gr}{v^2} = \mu$

$\frac{9.8(5)}{15.708^2} = \mu \Rightarrow \boxed{\mu = .1985}$

Conical Pendulum – (horizontal circular motion)
Constant speed

A particle of mass, *m,* is suspended from a string of length, *L,* and travels at a constant speed, *v* in a horizontal circle if radius, *r*. The string makes an angle, θ, given by sin θ = *r/L*, as shown in the figure.

a. Find the tension in the string and the speed of the particle.



$F_{T_y} = mg = F_T \cos\theta \Rightarrow \boxed{F_T = mg/\cos\theta}$

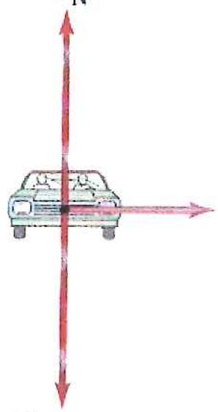
$F_c = F_{Tx} = \frac{mv^2}{r} = F_T \sin\theta$

$\frac{\cancel{m}v^2}{r} = \frac{\cancel{m}g}{\cos\theta} \cdot \sin\theta$

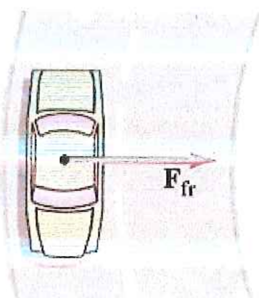
$v^2 = rg\tan\theta$

$\boxed{v = \sqrt{rg\tan\theta}}$

FIGURE 5–13 Forces on a car rounding a curve on a flat road, Example 5–7. (a) Front view, (b) top view.



(a)



(b)

EXAMPLE 5-7 (from book)

A Car rounding a bend

A 1000 kg car rounds a curve on a flat road of radius 50 m at a speed of 50 km/hr (14 m/s). Will the car make the turn, or will it skid, if: (a) the pavement is dry and the coefficient of static friction is $\mu_s = 0.60$; (b) the pavement is icy and $\mu_s = 0.25$?

$m = 1000 kg$

$r = 50 m$

$v = 14 m/s$

a. $\mu_s = .6$

$F_c = F_f = \mu F_N = \frac{mv^2}{r}$

$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r}$

$.6(9.8) = \frac{v^2}{50}$

$v = 17.14 m/s > 14 m/s$

$\boxed{\text{Yes,}}$ will make the turn.

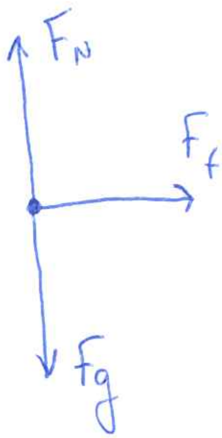
b. $\mu_s = .25$

$\mu m g = \frac{mv^2}{r}$

$.6(9.8) = \frac{v^2}{50} \rightarrow v = 11.07 m/s < 14 m/s$

$\boxed{\text{No,}}$ the car will slide.

11. (II) What is the maximum speed with which a 1050-kg car can round a turn of radius 70 m on a flat road if the coefficient of friction between tires and road is 0.80? Is this result independent of the mass of the car?



$$F_c = F_f = \frac{mv^2}{r}$$

$$\mu F_N = \frac{mv^2}{r}$$

$$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r}$$

$$.8(9.8) = \frac{v^2}{70}$$

$$\boxed{V = 23.426\ m/s}$$